A Study of Deformation Mechanism in Granular Materials Based on SlipLine Analyses(すべり線解析に基づく粒状体の変形機構の研究)

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論　文　内　容　要　旨

CHAPTER 1
INTRODUCTION

The correct formulation of constitutive equations to describe the mechanical behavior of granular materials in macroscopic and microscopic scales is one of outstanding problems in continuum mechanics. If attention is confined to plane deformation, then the failure hypothesis originally proposed by Coulomb (1773) provides a basis for the determination of the stresses in granular materials. However, the formulation of equations which determine its deformation is a more difficult and controversial problem even in the case of plane deformations. Many theories have been proposed, but none of them has found general acceptance.

An important feature of the behavior of sand is that, dense sand under low confining pressure show a peak in a shear stress-shear strain curve, followed by softening as the sand dilates. The existence of this softening behavior can result in the formation of the so-called shear bands. The analysis of this discontinuous behavior has been the subject of much research and debate. It is of importance both in the laboratory, where shear bands complicate the interpretation of so-called homogenous tests, and also in the field, where the occurence of shear bands poses particular difficulties in the scaling of laboratory test results. On the other hand, the actual value of the shear band inclination has received a little attention until now. Some researchers suggested that direction is corresponding to Rankine's theory, while others suggested it is corresponding to Roscoe's theory.

This study is initiated for the purpose of a better understanding of the deformation mechanism of granular materials through different slip-line analyses with emphasize on a micromechanically based constitutive model in which both contributions due to fabric changes and slippages among particles are incorporated. Also, the actual value of the shear band orientation (as an example of bifurcation problem) has been clarified which has a great effect on the earth pressure problems such as the determination of bearing capacity of footings.

CHAPTER 2
SLIP-LINE ANALYSIS OF GRANULAR MATERIALS USING OBLIQUE COORDINATE SYSTEM

There are numerous theories in the past dealt with the deformation mechanism of granular materials. Among those theories, it is found that the double-shearing model first proposed by Spencer (1964) and modified by Mehrababi and Cowin (1978) for dilatant granular materials seems to be more realistic. The model incorporates the facts that stress and deformation rate are not in general coaxial, and that the velocity and stress characteristis coincide. The model also depends on the mechanism proposed by Butterfield and Harkness (1972) in which shearing is accompanied by a stretch in the direction normal to the shear plane. Mehrabadi-Cowin velocity equations are given as

$$\begin{aligned} \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} &= \frac{\sin \nu}{\cos(\phi-\nu)} \left[\left(\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y} \right) \cos 2\Psi + \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) \sin 2\Psi \right] \\ \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} + 2\dot{\Psi} &= \frac{\cos \nu}{\sin(\phi-\nu)} \left[\left(\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y} \right) \sin 2\Psi \, 0 - \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_x}{\partial x} \right) \cos 2\Psi \right] \end{aligned} \qquad (1)$$

where, the material parameters of angle of internal friction and angle of dilatancy are denoted by ϕ, ν, respectively. Ψ is the inclination of the maximun principal stress to the x-axis and the superimposed dot is the material time-rate.

Equation (1) is derived by referring the velocity components (v_x, v_y) to a cartesian coordinate system (x, y). As it is well known, both the stress and velocity characteristics for granular materials are included by an obtuse angle ($\pi / 2 + \phi$) not a right angle as in metals, therefore, it seems reasonable to refer the velocity components to an pblique coordinate system (ξ, η), then Eq. (1) becomes

$$\frac{\partial v_\xi}{\partial \zeta}+\frac{\partial v_\eta}{\partial \eta}=\frac{\sin \nu}{\cos(\phi-\nu)}\left(\frac{\partial v_\xi}{\partial \eta}+\frac{\partial v_\eta}{\partial \xi}\right)$$

$$\frac{\partial v_\xi}{\partial \xi}-\frac{\partial v_\eta}{\partial \eta}=\frac{\sin \nu}{\cos(\phi-\nu)}\left(\frac{\partial v_\eta}{\partial \xi}-\frac{\partial v_\xi}{\partial \eta}\right) \tag{2}$$

As it can be seen from Eq. (2), introducing oblique coordinate system leads to considerable mathematical simlicity compared with using cartesian one.

Also, in this chapter, an expression is derived relative to oblique coordinate which enables the rate of energy dissipation $\dot{W}$ to be calculated for any pair of stress and velocity fields, this is specialised to Mehrabadi-Cowin model and a kinematic inequality are obtained as

$$\dot{W}=\left(q-\frac{p \sin \nu}{\cos(\phi-\nu)}\right)\frac{tr(\sigma' \cdot d')}{q}$$

$$\frac{\partial v_\xi}{\partial \xi}+\frac{\partial v_\eta}{\partial \eta}+\frac{1}{\cos \phi}\left[\frac{(v_\xi-v_\eta \sin \phi)}{\rho_\xi}-\frac{(v_\eta-v_\xi \sin \phi)}{\rho_\eta}\right] \geqq 0 \tag{3}$$

where p, q, σ, d are the mean stress, deviatoric stress, deviatoric stress tensor, and strain-rate tensor, respectively. ρ_ξ, ρ_η denote the stress gradient $\partial \Psi / \partial \xi$, $\partial \Psi / \partial \eta$.

CHAPTER 3
APPLICATION OF SLIP-LINE ANALYSIS TO PLANE PROBLEMS

Many solutions of the stress equations described in Chapter 2 , have been obtained. However, no attempt has been made to apply Mehrabadi-Cowin's velocity equations to any soil mechanics problems to determine the velocity field and to study the effect of the dilatancy concept on the deformation mechanism of granular materials, which is the purpose of this chapter.

Three different plane strain problems related to practical application are considered as follows:

(I) Shear of a long slab of granular material between two rigid, parallel rough plates. This case is related to geophysical problem, for example, rock strata in the earth contain joint and fractures which become filled with broken material and detritus, if such a jointed rock mass be subjected to stress other than hydrostatic pressure,then relative motion of the strata on either side of the discontinuity may occur. The velocity components (v_ξ, v_η) are determined and an expression for the rate of energy dissipation is derived. It is found that, the deformation region is not uniquely determind.

(II) Compression of a long slab of granular material between two rigid, parallel rough plates. This case is a model for the problem of a raft foundation supporting a building on a thiner layer of soft soil which covers rigid sustrata. An analytical solution of the velocity field is derived enables solutions to be found in which $\dot{\Psi} \fallingdotseq 0$ are supplies and strategy for obtaing stress fields with a compatible velocity field.

(Ⅲ) Stress time-dependent solution for shear flow of granular materials which can be related to the flow of the materials through hopper or chute. It is noted that, the direction of rotation of the principal axes is always away from the steady solution (usually assumed in that kind of problems), and according to the theory, the steady solution seems to be inherently unstable.

CHAPTER 4
A NEW STRESS-DILATANCY MODELING BASED ON MICROSCOPICAL APPROACHES

Considering the particulate media to be a continuum, it is possible to analyse complex and heterogeneous problems. To date, no entirely satisfactory constitutive model for granular material exists due to its complex macroscopic behavior. It is, therefore, instructive to investigate the deformation processes which may occur in particulate assemblies. Many theoretical studies of shear deformation of granular materials such as sand have performed from a standpoint that sand is composed of discrete grains. One group of these theories is called stress dilatancy theories which the author has especial interest in this chapter.

By restricting the attention to the deformation behavior of granular materials under simple shear condition, a particle model for two-dimension deformation is proposed for the derivation of a stress-dilatancy equation, which introduces no additional kinematical or dynamical parameters than have been presented by other researchers in the literature, nevertheless, it explains in a simple and convincing manner the phenomena of initial densification and subsequent dilatancy. The stress-dilatancy equation is given as

$$\frac{d\varepsilon_v}{d\gamma}=\frac{\tan\phi_\mu-\tau/\sigma}{1+\tan\phi_\mu\cdot\tau/\sigma} \tag{4}$$

where $d\varepsilon_v$, $d\gamma$ are the increments of volumetric and shear strains, respectively. ϕ_μ denotes the interparticle friction angle of the material.

Also, in this chapter, a microstructural approach in three-dimension is presente in which the behavior of an macro-system made of many representative micro-systems can be estimated from the behavior of an individual micro-system through ensemble average. The deformation of a sample is governed by the shear-normal stress ratio on the mobilized plane. Based on this approach, a constitutive relation is derived under cubical triaxial test condition which is given as

$$d\varepsilon_i=\lambda\cdot s_i/\sqrt{\sigma_i} \qquad (i=1,2,3) \tag{5}$$

where $d\varepsilon_i$, s_i, σ_i denote strain rate, unit slidig vector, principal stress, respectively. The parameter λ is linearly proportional to the magnitude of sliding movement. It is found that, it is possible to obtain a flow rule purely from the principle of sliding mechanism without using additional assumptions on potential function. Depending on Eq. (5), a three-dimensional isotropic stress-dilatancy equation is derived as

$$-\frac{d\varepsilon_v}{d\varepsilon_1}=\frac{3K\sigma_1(\tau_N/\sigma_N)\tan(\phi_\mu-\phi_m)}{(\sigma_N-\sigma_1)-\tau_N\tan(\phi_\mu-\phi_m)} \tag{6}$$

where ϕ_m is the mobilized angle of friction.

The integrity of the proposed stress-dilatancy equations (4) and (6) is examined by different existing experimental data. It is found that, the estimated values agree satisfactorily well with the experimental results.

Chapter 5
DERIVATION OF SHEAR BAND ORIENTATION FROM PLASTIC FLOW THEORY

To clarify the actual value of the shear band inclination, an elastic plastic constitutive relation is derived for dilatant, pressure sensitive, and work hardening granular material undergoing rate-independent single slip in which each slip plane is assumed to harden independently. The constitutive relation is written as

$$\overset{\nabla}{\sigma}_{ij}=L_{ijkl}D_{kl} \tag{7}$$

where $\overset{\nabla}{\sigma}_{ij}$, D_{kl} denote Jaumann rate of Cauchy stress, strain tensors, repectively. L_{ijkl} is the elastic-plastic modulus tensor. It is found that, the principal directions of the stress and deformation rate tensors do not coincide, thus the theory results in non-coaxiality, inspite of the assumption that the material is elastically isotropic.

By considering the initiation of shear band formation as an instability of plastic flow, using Eq. (7) and by means of the bifurcation theory, the inclination of the shear band (θ) to the minor principal stress direction σ_3, is given by

$$\theta=\pm\arctan\left[\frac{2(1-MB)+\hat{\nu}(3B-M-1)}{2(1-M)(1-B)}\right]^{1/2} \tag{8}$$

where M,B are the pressure-sensitivity and dilatancy factors, respectively. $\hat{\nu}$ denotes Poisson's ratio. Equation (8) is approximated as

$$\theta=\pm(\pi/4+\xi/2)=\pm[\pi/4+(\phi+\nu)/4] \tag{9}$$

where $\xi\simeq(\phi+\nu)/2$ is assumed as a material constant different from ϕ which determines the direction of slip-lines.

It is found that, the comparison between the theoretical prediction of the shear band orientation of Eq. (8) with its approximation of Eq. (9) and different existing experimental data, shows a remarkably good agreement.

CHAPTER 6
ANALYSIS OF BEARING CAPACITY OF STRIP FOOTING

The bearing capacity of strip footings is generally calculated using Terzaghi's (1943) equation in which the dimensionless bearing capacity factors N_c, N_q, and N_γ due to cohesion (c), surface surcharge (q_s), and unit weight (γ) of the material, are available in tables or charts and assumed to be functions of the soil friction angle only. Terzaghi's values are frequently used in practice, but many other values exist, and in the case of N_γ these can vary greatly. It is concluded that most of the attempts for calculating bearing capacity depend on the classical assumption that the inclination of the slip lines are $\pm(\pi/4+\phi/2)$ to the minor principal stress direction. Hence, in carrying out conventional stability analysis in which a failure plane is assumed, it is desirable to think of the failure surfaces to be inclined by an angle $\pm(\pi/4+(\phi+\nu)/4)$ to the minor principal stress direction (depending on the findings from Chapter 5), rather than to be the planes of maximum stress ratio. Based on this assumption, an analytical formulation for computing bearing capacity of a smooth strip footing is derived as

$$\sigma_{ult} = cN_c + q_sN_q + 0.5\gamma bN_\gamma \tag{10}$$

where N_c, N_q, and N_γ are dimensionless bearing capacity factors and b is the semi-width of the footing. It is noted that, the two factors N_c and N_q are nearly equal to Terzaghi's factors, while the N_γ factors is approximetely 50% of Terzaghi's one.

Also, in this chapter, a numerical solution for predicting bearing capacity is developed using finite difference technique. It is found that, both the analytical and numerical solutions are in good agreement with different existing experimental data.

CHAPTER 7
GENERAL CONCLUSIONS

Assuming granular material obeys Mohr-Coulomb yield criterion, it is found that, by introducing an oblique cartesian system in deriving both of the stress and velocity equations proposed by Mehrabadi-Cowin leads to considerable mathematical simplicity compared with using cartesian one. By applying Mehrabadi-Cowin theory for different plane strain problems, it is found that, the dilatancy has remarkable effect on the velocity field distribution which can not be neglected and also, the direction of rotation of the principal axes is always away from the steady state solution.

Two and three-dimensional stress dilatancy equations are derived based on microscopical approaches in which the slip directions are away from that given by Coulomb's solution. The predicted values from these equations agree well with many existing experimental data.

An expression for the direction of shear band orientation is derived based on a micromechanically based constitutive model in which bifurcation of the deformation is possible. The estimated values are in good agreement with many existing experimental results.

An analytical equation and a numerical solution for predicting the ultimate bearing capacity of a surface smooth strip footing under plane strain condition are derived taking into account the suggested slip direction. The predicted bearing capacity by the two approaches lead to a reasonable result which is found to agree with other published experimental data.

Finally, it can be concluded that, the proposed slip direction may provide an alternative way towards the objective of better understanding the deformation mechanism in granular materials.

審 査 結 果 の 要 旨

土や砂のような粒状体は，通常の連続固体にはみられない複雑な変形挙動を示し，その変形機構の解明が求められている。本論文は，すべり線の解析によって，粒状態の変形・流動の性質を研究するとともに，ダイレイタンシィ特性やせん断帯の形成について考察を行ったもので，全編7章よりなる。

第1章は序論であり，本研究の背景について述べている。

第2章では，粒状体のすべり線の斜交性に着目し，斜交座標系を用いる解析を提示している。とくに，ダイレイタンシィを考慮した速度場の方程式が，この解析によって容易に導き出されることを示し，散逸エネルギーの表現や速度場の不連続性などについて考察を加えている。

第3章は，第2章で得られた速度場の方程式の適用例を示したものである。平行板にはされまた粒状体の流れ等3つの問題に応用し，従来行われていたダイレイタンシィを考慮しない非圧縮性解析と比較を行って，ダイレイタンシィの影響等を明らかにしている。

第4章では，粒状体力学の一つの基礎式である応力・ダイレイタンシィ式について考察し，理論式の提案を行っている。単純せん断と三軸試験の場合について，微視的変形機構に基づいて考察し，3次元の場合には空間滑動面を応用して新しい流動則を導出している。さらに，ここで得られた理論式は，実験結果ともよく適合することを示している。これらは重要な知見である。

第5章では，粒状体におけるせん断帯の形成とその傾斜角について考察している。従来の二重すべり理論ではなく，微視的考察から導いた関係式の分岐条件から，せん断帯の傾斜角を求め，他の理論に比較して実験値との適合性がよいことを示している。これは新しい知見である。

第6章ではフーティングの支持力計算には，前章で得られた傾斜角を応用して解析する手法を提示している。計算結果は実測値ともよく対応することを例示し，手法の妥当性を示している。

第7章は結論である。

以上要するに本論文は，すべり線の解析によって粒状体の変形機構を系統的に考察し，新しい多くの知見を得たもので，粒状体力学，土木材料力学の発展に寄与する処が少なくない。

よって，本論文は工学博士の学位論文として合格と認める。